

# FOLSOM CORDOVA UNIFIED SCHOOL DISTRICT

## EAP SENIOR YEAR MATHEMATICS (ESM) COURSE

**Date:** January 2017

**Proposed Grade Level(s):** 12<sup>th</sup>

**Grading:** A-F

**Prerequisite(s):** Completion of Integrated Math 3 and Math EAP score of 2 or 3

**Intent to Pursue 'A-G' College Prep Status:** Yes

**Course Length:** 1 year

**Subject Area:** Mathematics

**Credits:** 5 per semester

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### **COURSE DESCRIPTION:**

This course is designed to strengthen student's mathematical foundation and prepare them to be college and career ready. The goal of the course is to deepen conceptual understanding of mathematical theory, skills, and strategies. The course is designed to incorporate National Common Core Standards for Mathematical Practice and is aligned with specific high school standards listed in the California Common Core State Standards for Mathematics (CCSS-M). Utilizing practical life applications,, this course serves both college and career bound high school seniors.

Currently a significant percentage of students are required to take one or more remedial math courses upon entering college, delaying their entry into college level math, and possibly their graduation date. The purpose of this course is to fulfill the need to provide more math options that support transition to college. With a focus on depth not breadth, students can master mathematical content and be able to transfer their skills to college and to career pathways.

### **GENERAL GOALS/ESSENTIAL QUESTIONS:**

#### **Goals:**

- Develop a growth mindset towards mathematics that enables the student to continue to persevere through problem solving in higher level math courses.
- Become better problem solvers.
- Build critical thinking skills.
- Increase their perseverance to make sense of and to solve real-world and mathematical problems.
- Deepen their understanding of underlying structures of mathematics.
- Gain appreciation of mathematics and its applications.
- Improve their ability to communicate their mathematical thinking.
- Develop their ability to work effectively as a member of a team.

#### **Essential Questions:**

- Do students demonstrate the Standards for Mathematical Practice when engaged in mathematics?
- Do students flexibly apply problem solving strategies (e.g., guess and check, logic/deductive reasoning, tables and lists,...) to contextual situations to deepen conceptual understandings of the structures and applications of mathematics?
- Do students analyze the information imbedded in different types of contextual problems and determine what data is given and what assumptions can be justified?

- Do students identify and assess the importance of ambiguities and complexities within a problem?
- Have students strengthened their number sense and procedural fluency?
- Do students make connections between numeric and algebraic expressions and representations?
- Do students examine and apply families of functions including: linear, quadratic, exponential, logarithmic, absolute value, and piece-wise?
- Do students make connections between conceptual categories of mathematical content?
- Do students utilize understanding of linear and exponential functions in financial contexts?
- Do students reflect on their work and edit for clarity and accuracy?
- Do students make generalizations based on observations and repeated reasoning.
- Do students communicate reasoning verbally and in writing?
- Do students provide an appropriate level of justification in an organized viable argument, free from logical and arithmetical errors?
- Do students work collaboratively within small groups?

## **COMMON CORE STATE ANCHOR STANDARDS FOR READING (K-12):**

### **Key Ideas and Details**

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, or ideas develop and interact over the course of a text.

### **Craft and Structure**

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

### **Integration of Knowledge and Ideas**

7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

### **Reading Range/Text Complexity**

10. Read and comprehend complex literary and informational texts independently and proficiently.

## **COMMON CORE STATE ANCHOR STANDARDS FOR WRITING (K-12):**

### **Text Types and Purposes**

1. Write arguments to support claims in an analysis of substantive topics or texts using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details and well-structured event sequences.

### **Production and Distribution of Writing**

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

### **Research to Build Knowledge**

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

### **Range of Writing**

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

## **COMMON CORE STATE ANCHOR STANDARDS FOR SPEAKING AND LISTENING (K-12):**

### **Comprehension and Collaboration**

1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

### **Presentation of Knowledge and Ideas**

4. Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and ensure that the organization, development, and style are appropriate to task, purpose, and audience.
5. Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
6. Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

## **CTE INDUSTRY SECTOR / PATHWAY / STANDARDS:**

N/A

## **DETAILED UNITS OF INSTRUCTION:**

### **Unit 1: Team Building and Problem Solving**

Students gain insight into the applications of a variety of problem solving strategies. Each problem requires that students engage in multiple standards of mathematical practice as outlined in the CCSS-M. In addition, students will develop team building skills to work well in groups and enhance their mathematical communication skills. This unit sets the climate for the rest of the course, stressing skills such as reasoning and

perseverance in traditional as well as nontraditional problems. These types of problems and strategies will be used throughout the course.

## **Unit 2: Linear Functions**

This is the first of several units in which students will revisit concepts learned in prior mathematics courses through a different lens, resulting in a deeper understanding of the concepts, as well as the ability to apply their learning to real situations. Physical models will be used to illustrate the concepts of rate of change (slope) and the  $0^{th}$  term (y-intercept), and students will transition to understanding more abstract models. Students will determine what makes a function linear or nonlinear and use functions to make predictions. Function notation will be used as well as iterative and recursive notation.

Basic operations with functions will also be addressed. Real numbers, fractions, decimals, and percent are all practiced within the sequences and resulting explicit equations. Students interpret and derive algebraic representations (including graphs) of linear equations and make decisions regarding which form is most useful in a given context.

## **Unit 3 Quadratic Functions**

Students identify the key components unique to quadratic equations, distinguish them from other equations, and solve problems that utilize quadratics in both physical models and business applications. Students will also recognize and compare local maxima and/or minima within various business models, and must draw conclusions and explain their meaning and impact to the business. Students will apply their understanding of equations in a catapult project. The unit concludes with an introduction to complex numbers in context of quadratic functions that have imaginary solutions.

## **Unit 4: Exponential Functions**

This unit emphasis is placed on real world applications. The math developed here supports a future unit, the Math of Finance. Numeracy is built by using the concept that every number is 100% of itself. The unit begins with concrete lessons, using manipulatives to demonstrate growth and decay, and progresses to more abstract situations. Transformation is explored cursorily.

## **Unit 5: Systems of Equations & Inequalities**

Development of a deeper understanding of what the solution to a system of equations and inequalities would represent is the focus of this unit. The students will explore two functions and their input-output tables through graphing to determine that the solution represents where one input yields the same output for two separate functions. Once students become conscious of what a solution to a system is, they will have additional real world supply and demand problems and explore actual decision making questions based on the graph and solution. Finally the students will explore linear programming which includes inequalities (constraints) and a profit equation (objective function).

## **Unit 6: Logarithmic Functions**

The lessons in this unit were designed to demystify logarithms and build a strong foundation, increasing students' knowledge and confidence in their ability to solve or define logarithms.

## **Unit 7: Absolute Value and Piecewise Functions**

Students will explore, in depth, the concept of absolute value through algebraic representations and graphical representation of functions' absolute values. The students will also solve and graph absolute value inequalities and be able to decompose an absolute value function into an equivalent piecewise function. The importance of piecewise functions will be explored using real world data.

## **Unit 8: Financial Math**

The math related to finance is a new topic for most students. Practical applications and intricate formula derivation will be used for their adult life decisions. This unit is only an introduction and does not detail all the complex points of the financial subject.

### **TEXTBOOKS AND RESOURCE MATERIALS:**

**Textbook** *California State University Sacramento EAP Senior Year Mathematics (ESM) Course Materials*

### **Resource Materials**

ESM Training and Support

### **SUBJECT AREA CONTENT STANDARDS TO BE ADDRESSED:**

The eight *Standards for Mathematical Practice* describe the attributes of mathematically proficient students and expertise that mathematics educators at all levels should seek to develop in their students. Mathematical practices provide a vehicle through which students engage with and learn mathematics with a focus on reading, writing, and explaining.

All eight standards for mathematical practice are integrated repeatedly throughout this course.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The California Math Content Standards were designed to encourage the highest achievement of every student, by defining the knowledge, concepts, and skills that students should acquire at each grade level. The math content standards addressed in this course are as follows:

### **Number and Quantity**

#### **Number and Quantity:**

*Reason quantitatively and use units to solve problems.*

N-Q 1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q 2: Define appropriate quantities for the purpose of descriptive modeling.

N-Q 3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

#### **The Complex Number System:**

*Perform arithmetic operations with complex numbers.*

N-CN 1: Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.

N-CN 2: Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N-CN 3: Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

*Use complex numbers in polynomial identities and equations.*

N-CN 7: Solve quadratic equations with real coefficients that have complex solutions.

N-CN 8: Extend polynomial identities to the complex numbers.

## **Algebra**

### **Seeing Structure in Expressions**

*Interpret the structure of expressions.*

A-SSE 1: Interpret expressions that represent a quantity in terms of its context.

A-SSE 1b: Interpret complicated expressions by viewing one or more of their parts as a single entity.

A-SSE 2: Use the structure of an expression to identify ways to rewrite it.

*Write expressions in equivalent form.*

A-SSE 3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

### **Arithmetic with Polynomials and Rational Expressions**

*Understand the relationship between zeros and factors of polynomials.*

A-APR 3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

### **Creating Equations**

*Create equations that describe numbers of relationships.*

A-CED 1: Create equations and inequalities in one variable including ones with absolute value, and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED 2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED 3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

### **Reasoning with Equations and Inequalities**

*Understand solving equations as a process of reasoning and explain the reasoning.*

A-REI 1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

*Solve equations and inequalities in one variable.*

A-REI 3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI 4: Solve quadratic equations in one variable.

*Solve systems of equations.*

A-REI 5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI 6: Solve systems of linear equations exactly and approximately (e.g. with graphs), focusing on pairs of linear equations in two variables.

A-REI 7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

*Represent and solve equations and inequalities graphically.*

- A-REI 10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- A-REI 11: Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-REI 12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## **Functions**

### **Interpreting Functions**

*Understand the concept of a function and use function notation.*

- F-IF 1: Understand that a function from one set (called the domain) to another set, (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input of  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF 2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-IF 3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

*Interpret functions that arise in applications in terms of the context.*

- F-IF 4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF 5: Relate the domain of function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF 6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

*Analyze functions using different representations.*

- F-IF 7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- F-IF 7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-IF 7b: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- F-IF 7e: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F-IF 8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- F-IF 9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

### **Building Functions**

*Build a function that models a relationship between two quantities.*

- F-BF 1: Write a function that describes a relationship between two quantities.
- F-BF 2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

*Build new functions from existing functions.*

F-BF 3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ , and  $f(x+k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF 5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

### **Linear, Quadratic, and Exponential Models**

*Construct and compare linear, quadratic, and exponential models and solve problems.*

F-LE 1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE 2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

F-LE 3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F-LE 4: For exponential models, express as a logarithm the solution to  $ab^{ct} = d$ , where  $a, c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

F-LE 4.1: Prove simple laws of logarithms.

F-LE 4.2: Use the definition of logarithms to translate between logarithms in any base.

F-LE 4.3: Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

*Interpret expressions for function in terms of the situation they model.*

F-LE 5: Interpret the parameters in a linear or exponential function in terms of context.

F-LE 6: Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.

### **DISTRICT ESLRS TO BE ADDRESSED:**

#### **Students will be:**

- **Self-Directed Learners:** as they increase perseverance to make sense of and to solve real-world and mathematical problems.
- **Constructive Thinkers:** as they become better problem solvers and build critical thinking skills.
- **Effective Communicators:** as they improve their ability to communicate their mathematical thinking.
- **Collaborative Workers:** as they develop their ability to work effectively as a member of a team.
- **Quality Producers/Performers:** improve their number sense and procedural fluency, and develop their understanding of the underlying structure of mathematics.
- **Responsible Citizens:** by gaining an appreciation of mathematics and applications.