

FOLSOM CORDOVA UNIFIED SCHOOL DISTRICT

INTEGRATED MATH 3 FOUNDATIONS

Date: January, 2017

Proposed Grade Level(s): 9-12

Grading: A-F

CTE Sector / Pathway: N/A

Prerequisite(s): Completion of IM 2

Course Length: 1 year

Subject Area: Mathematics

Credits: 5/Semester

Intent to Pursue 'A-G' College Prep Status: No

COURSE DESCRIPTION:

Integrated Math 3 Foundations is a course designed for students who are not yet ready for the rigor and pace of Integrated Math 3. It aims to apply and extend what students have learned in previous courses by focusing on connections between multiple representations of functions, transformations of different function families, finding zeros of polynomials and connecting them to graphs and equations of polynomials, modeling periodic phenomena with trigonometry, and understanding the role of randomness and the normal distribution in making statistical conclusions.

Students use problem-solving strategies, questioning techniques, investigation, critical analysis, gathering and constructing evidence, and communicating rigorous arguments, to justify their thinking. Students learn in collaboration with others while sharing information, expertise, and ideas. The course is well balanced between procedural fluency (algorithms and basic skills), deep conceptual understanding, strategic competence (problem solving), and adaptive reasoning (extension and transference). The course imbeds the CCSS Standards for Mathematical Practice as an integral part of the lessons.

Upon successful completion of this course, students should have the necessary mathematical skills to start Integrated Math 3.

Key concepts addressed in this course are:

- Visualize, express, interpret and describe, and graph functions and their inverses. Given a graph, students will be able to represent the function with an equation, and vice-versa, and transform the graph, including the following function families:
 - absolute value
 - exponential
 - linear
 - logarithmic
 - piecewise-defined
 - polynomial
 - quadratic
 - square root
 - trigonometric
- Use of variables and functions to represent relationships given in tables, graphs, situations, and geometric diagrams, and recognize the connections among these multiple representations.

- Application of multiple algebraic representations to model and solve problems presented as real world situations or simulations.
- Solving linear or quadratic equations in one variable, and systems of equations in two variables.
- Use of algebra to rewrite complicated algebraic expressions and equations in more useful forms.
- Rewriting rational expressions and arithmetic operations on polynomials.
- The relationship between zeros and factors of polynomials.
- Operations with complex numbers, and solving quadratics with complex solutions.
- Applications of the Law of Sines and Law of Cosines.
- Modeling periodic phenomena with trigonometric functions.
- Calculating the sums of arithmetic and geometric series, including infinite geometric series.
- Concepts of randomness and bias in survey design and interpretation of the results.
- Use of a normal distribution to model outcomes and to make inferences as appropriate.
- Use of computers to simulate and determine complex probabilities.
- Use of margin of error and sample-to-sample variability to evaluate statistical decisions.
- Solving trigonometric equations and proving trigonometric identities

GENERAL GOALS/ESSENTIAL QUESTIONS:

Goals:

- Students will build conceptual understanding of the mathematics.
- Students will establish connections among different concepts.
- Students will use strategic problem solving skills and collaboration to work on non-routine problems.
- Students will justify their reasoning, communicate their thinking, and generalize patterns.
- Students will be ready for the rigor and rich content of Integrated Math 3.

Essential Questions:

- Can the student visualize, express, interpret and describe, and graph functions and their inverses?
- Can the student use a multitude of graph families, to represent the function with an equation, and vice-versa, and transform the graph, including the following function families?
- Can the student solve linear and quadratic equations and systems of equations in two- to three-variable formats?
- Can the student use variables and functions to represent relationships given in tables, graphs, situations, and geometric diagrams, and recognize the connections among these multiple representations?

COMMON CORE STATE ANCHOR STANDARDS FOR READING (K-12):

Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, or ideas develop and interact over the course of a text.

Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.

5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

Reading Range / Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

COMMON CORE STATE ANCHOR STANDARDS FOR WRITING (K-12):

Text Types and Purposes

1. Write arguments to support claims in an analysis of substantive topics or texts using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

COMMON CORE STATE ANCHOR STANDARDS FOR SPEAKING AND LISTENING (K-12):

Comprehension and Collaboration

1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.

- Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

Presentation of Knowledge and Ideas

Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and ensure that the organization, development, and style are appropriate to task, purpose, and audience.

- Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
- Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

CTE INDUSTRY SECTOR / PATHWAY / STANDARDS:

N/A

DETAILED UNITS OF INSTRUCTION:

CHAPTERS	SECTIONS
Chapter 1: Investigations and Functions	1.1.1 Solving a Function Puzzle in Teams 1.1.2 Using a Graphing Calculator to Explore a Function 1.1.3 Function Investigation 1.1.4 Combining Linear Functions
Chapter 2: Transformations of Parent Graphs	2.1.1 Transforming Quadratic Functions 2.1.2 Modeling with Parabolas 2.2.1 Transforming Other Parent Graphs 2.2.2 Describing (<i>hike</i>) for Each Family of Functions 2.2.3 Transformations of Functions 2.2.4 Transforming Non-Functions 2.2.5 Developing a Mathematical Model 2.3.1 Completing the Square
Chapter 3: Solving Inequalities	3.1.1 Strategies for Solving Equations 3.1.2 Solving Equations Graphically 3.1.3 Multiple Solutions to Systems of Equations 3.1.4 Using Systems of Equations to Solve Problems 3.2.1 Solving Inequalities with One or Two Variables 3.2.2 Using Systems to Solve a Problem 3.2.3 Applications of Systems of Inequalities 3.2.4 Using Graphs to Determine Solutions
Chapter 4: Normal Distributions and Geometric Modeling	4.1.1 Survey Design 4.1.2 Samples and the Role of Randomness 4.1.3 Bias in Convenience Samples 4.2.1 Testing Cause and Effect with Experiments 4.2.2 Conclusions from Studies 4.3.1 Relative Frequency Histograms 4.3.2 The Normal Probability Density Function 4.3.3 Percentiles 4.4.1 Cross-Sections and Solids of a Revolution 4.4.2 Modeling with Geometric Solids 4.4.3 Designing to Meet Constraints
Chapter 5: Inverses and Logarithms	5.1.1. “Undo” Equations

	<ul style="list-style-type: none"> 5.1.2 Using a Graph to Find an Inverse 5.1.3 More Inverse Functions 5.2.1 The Inverse of an Exponential Function 5.2.2 Defining the Inverse of an Exponential Function 5.2.3 Investigating the Family of Logarithmic Functions 5.2.4 Transformations of Logarithmic Functions
Chapter 6: Simulating Sampling Variability	<ul style="list-style-type: none"> 6.1.1 Simulations of Probability 6.1.2 More Simulations of Probability 6.1.3 Simulating Sampling Variability 6.2.1 Statistical Test Using Sampling Variability 6.2.2 Variability in Experimental Results 6.2.3 Quality Control 6.3.1 Analyzing Decisions and Strategies
Chapter 7: Logarithms and Triangles	<ul style="list-style-type: none"> 7.1.1 Using Logarithms to Solve Exponential Equations 7.1.2 Investigating the Properties of Logarithms 7.1.3 Writing Equations of Exponential Functions 7.1.4 An Application of Logarithms 7.2.1 Determining Missing Parts of Triangles 7.2.2 Law of Sines 7.2.3 Law of Cosines (optional) 7.2.4 The Ambiguous Case (optional) 7.2.5 Choosing a Tool (optional)
Chapter 8: Polynomials	<ul style="list-style-type: none"> 8.1.1 Sketching Graphs of Polynomial Functions 8.1.2 More Graphs of Polynomial Functions 8.1.3 Stretch Factors for Polynomial Functions 8.2.1 Writing Equations Using Complex Roots 8.2.2 More Real and Complex Roots 8.3.1 Polynomial Division 8.3.2 Factors and Rational Zeros 8.3.3 An Application of Polynomials 8.3.4 Special Cases of Factoring
Chapter 9: Trigonometric Functions	<ul style="list-style-type: none"> 9.1.1 Introductions to Periodic Models 9.1.2 Graphing the Sine Functions 9.1.3 Unit Circle \leftrightarrow Graph 9.1.4 Graphing and Interpreting the Cosine Function 9.1.5 Defining a Radian 9.1.6 Building a Unit Circle 9.1.7 The Tangent Function (optional) 9.2.1 Transformations $y = \sin(x)$ (optional) 9.2.2 One More Parameter for a Periodic Function (optional) 9.2.3 Period of a Trigonometric Function (optional) 9.2.4 Graph \leftrightarrow Equation (optional)
Chapter 10: Series	<ul style="list-style-type: none"> 10.1.1 Introduction to Arithmetic Series 10.1.2 More Arithmetic Series 10.1.3 General Arithmetic Series 10.1.4 Summation Notation and Combinations of Series 10.1.5 Mathematical Induction 10.2.1 Geometric Series 10.2.2 Infinite Series

	10.3.1 Using a Binomial Probability Model (optional) 10.3.2 Pascal’s Triangle and Binomial Theorem (optional) 10.3.3 The number e (optional)
Chapter 11: Rational Expressions and Three-Variable Systems	11.1.1 Simplifying Rational Expressions 11.1.2 Multiplying and Dividing Rational Expressions 11.1.3 Adding and Subtracting Rational Expressions 11.1.4 Operations with Rational Expressions 11.2.1 Creating a Three-Dimensional Model (optional) 11.2.2 Graphing Equations in Three Dimensions (optional) 11.2.3 Solving Systems of Three Equations with Three variables (optional) 11.2.4 Using Systems of Three Equations for Curve Fitting (optional)
Chapter 12: Analytic Trigonometry (Optional)	12.1.1 Analyzing Trigonometric Equations 12.1.2 Solutions to Trigonometric Equations 12.1.3 Inverses of Trigonometric Functions 12.1.4 Reciprocal Trigonometric Functions 12.2.1 Trigonometric Identities 12.2.2 Proving Trigonometric Identities 12.2.3 Angle Sum and Difference Identities
CHECKPOINT MATERIALS	Checkpoint 2: Solving Quadratic Equations Checkpoint 3: Function Notation and Describing a Function Checkpoint 4: Expressions with Integer and Rational Exponents Checkpoint 5: Transformations of Functions Checkpoint 6: Solving Complicated Equations and Systems Checkpoint 7: Solving and Graphing Inequalities Checkpoint 8: Determining the Equation for the Inverse of a Function Checkpoint 9A: Solving Equations with Exponents Checkpoint 9B: Rewriting Expressions and Solving Equations with Logarithms Checkpoint 10: Solving Triangles Checkpoint 11: Roots and Graphs of Polynomial Functions Checkpoint 12: Periodic Functions

TEXTBOOKS AND RESOURCE MATERIALS:

Textbook

Core Connections Integrated III by Kysh, Dietiker, Sallee, Hoey, CPM, (2015)

Resource Materials

***KUTA* software**

SUBJECT AREA CONTENT STANDARDS TO BE ADDRESSED:

The eight *Standards for Mathematical Practice* describe the attributes of mathematically proficient students and expertise that mathematics educators at all levels should seek to develop in their students. Mathematical practices provide a vehicle through which students engage with and learn mathematics with a focus on reading, writing, and explaining.

All eight standards for mathematical practice are integrated repeatedly throughout this course.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The California Math Content Standards were designed to encourage the highest achievement of every student, by defining the knowledge, concepts, and skills that students should acquire at each grade level. The math content standards addressed in this course are as follows:

Number and Quantity

The Complex Number System

Use complex numbers in polynomial identities and equations.

N-CN 8: Extend polynomial identities to the complex numbers.

N-CN 9: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Algebra

Seeing Structure in Expressions

Interpret the structure of expressions.

A-SSE 1: Interpret expressions that represent a quantity in terms of its context.

A-SSE 2: Use the structure of an expression to identify ways to rewrite it.

Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

A-APR1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

A-APR 2: Know and apply the Remainder Theorem: for a polynomial $p(x)$ and a number a , the remainder on division by $x-a$ is $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A-APR 3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

A-APR 4: Prove polynomial identities and use them to describe numerical relationships.

A-APR 5: Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions.

A-APR 6: Rewrite simple rational expressions in different form; write $a(x)/b(x)$ in the form $a(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree $r(x)$ less than the degree $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-APR 7: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations

Create equations that describe numbers or relationships.

A-CED 1: Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.

A-CED 2: Create equations in two or more variables to represent relationships between quantities.

A-CED 3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

A-CED 4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI 2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-REI 11: Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Functions

Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F-IF 4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF 5: Relate the domain of function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF 6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

F-IF 7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-IF 8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-IF 9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Building Functions

Build a function that models a relationship between two quantities.

F-BF 1: Write a function that describes a relationship between two quantities.

Build new functions from existing functions.

F-BF 3: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate

an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-BF 4: Find inverse functions.

Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE 4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$, where a, c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

F-LE 4.1: Prove simple laws of logarithms.

F-LE 4.2: Use the definition of logarithms to translate between logarithms in any base.

F-LE 4.3: Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle

F-TF 1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF 2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Geometry

Similarity, Right Triangles, and Trigonometry

Apply trigonometry to general triangles.

G-SRT 9: Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT 10: Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT 11: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.

G-GPE 3.1 Given a quadratic equation of the form $ax^2 + bx^2 + cx + dy + e = 0$, use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola and graph the equation.

Geometric Measurement and Dimension

Visualize relationships between two-dimensional and three-dimensional objects.

G-GMD 4: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry

Applying geometric concepts in modeling situations.

G-MG 1: Use geometric shapes, their measures, and their properties to describe objects.

G-MG 2: Apply concepts of density based on area and volume in modeling situations.

G-GM 3: Apply geometric methods to solve design problems.

Statistics and Probability

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

S-ID 4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

S-IC 1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S-IC 2: Decide if a specified model is consistent with results from a given data-generating process.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S-IC 3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S-IC 4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S-IC 5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S-IC 6: Evaluate reports based on data.

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions.

S-MD 6: Use probabilities to make fair decisions.

S-MD 7: Analyze decisions and strategies using probability concepts.

DISTRICT ESLRS TO BE ADDRESSED: give an example for each bulleted item

Students will be:

- **Self-Directed Learners:** as they increase perseverance to make sense of and to solve real-world and mathematical problems.
- **Constructive Thinkers:** as they become better problem solvers and build critical thinking skills
- **Effective Communicators:** as they improve their ability to communicate their mathematical thinking.
- **Collaborative Workers** as they develop their ability to work effectively as a member of a team
- **Quality Producers/Performers:** improve their number sense and procedural fluency, and develop their understanding of the underlying structure of mathematics.
- **Responsible Citizens:** by gaining an appreciation of mathematics and applications.