

FOLSOM CORDOVA UNIFIED SCHOOL DISTRICT

HONORS INTEGRATED MATH 3

Date: January, 2016

Proposed Grade Level(s): 9th-12th

Grading: A-F

Prerequisite(s): “A” or better in Integrated Math 2

Course Length: One year

Subject Area: Mathematics

Credits: 5 per semester

Articulation Units: N/A

Intent to Pursue ‘A-G’ College Prep Status: Yes

COURSE DESCRIPTION:

Honors Integrated Math 3 is designed to extend and apply the mathematics learned in previous math courses at an accelerated pace and incorporate Pre-Calculus topics to prepare students to enter Calculus. The standards are based on the Common Core State Standards for Mathematics and include topics from the conceptual categories: number and quantity, Algebra, functions, geometry, and statistics and probability. Instructional time will focus on six critical areas: (1) apply methods from probability and statistics to draw inferences and conclusions from data; (2) expand understanding of functions to include polynomial, rational, and radical functions; (3) expand right triangle trigonometry to include general triangles; (4) consolidate functions and geometry to create models and solve contextual problems; (5) expands upon the basic trigonometry first introduced in Integrated Math 2, but includes graphing trig functions, solving trig equations, and proving identities, as well as applying trigonometry with the Law of Sines, Law of Cosines, and finding the area of a triangle; and (6) an introduction to polar coordinates, graphs of polar equations, and vectors are also introduced in this course.

GENERAL GOALS/ESSENTIAL QUESTIONS:

As stated in the *Mathematics Framework (2013)*, the focus of Honors Integrated Math 3 is for students to expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to include general triangles. Finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. Mathematical modeling is a major theme of this course as it involves the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions. The courses in the Integrated Pathway follow the structure that began in the K-8 standards of presenting mathematics as a coherent subject, mixing standards from various conceptual categories. In addition to the math analysis content taught, the graphing, solving, and applications of trigonometry are taught in preparation of their use in calculus, as well as other advanced science and math courses.

CCSS STUDENT READING/WRITING/SPEAKING and LISTENING COMPONENTS:

The curriculum has literacy strategies embedded within the text that assists students in the following:

- Understand math tasks
- Communicating understanding orally and through writing
- Writing about math
- Building math vocabulary

- Building academic vocabulary

The eight *Standards for Mathematical Practice* describe the attributes of mathematically proficient students and expertise that mathematics educators at all levels should seek to develop in their students. Mathematical practices provide a vehicle through which students engage with and learn mathematics – with a heavy focus on reading, writing, and explaining.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

DETAILED UNITS OF INSTRUCTION:

Chapters	Standards	Chapter Overview
1: Interpreting Data in Normal Distributions	S.ID.1 S.ID.2 S.ID.4	The first lesson of this chapter leverages student knowledge of relative frequency histograms to introduce normal distributions. Students explore the characteristics of normal distributions. In the second lesson, students build their knowledge of normal distributions using the Empirical Rule for Normal Distributions. Students use the Empirical Rule for Normal Distributions is to determine the percent of data between given intervals that are bounded by integer multiples of the standard deviation from the mean. In the third lesson, students use a z-score table and a graphing calculator to determine the percent of data in given intervals that are bounded by non-integer multiples of the standard deviation from the mean. In the last lesson, students use their knowledge of probability and normal distributions to analyze scenarios and make decisions.
2: Making Inferences and Justifying Conclusions	S.IC.1 S.IC.2 S.IC.3 S.IC.4 S.IC.5 S.IC.6	The first two lessons focus on methods of collecting data to analyze a question or characteristic of interest, specific sampling methods, and the significance of randomization. Then, students use data from samples to estimate population means and proportions, and determine whether results are statistically significant. In the last lesson, students have the opportunity to complete a culminating project based on concepts from the chapter.
3: Searching for Patterns	A.SSE.1a A.SSE.1b A.SSE.2	This chapter begins with opportunities for students to analyze and describe various patterns. Questions ask students to represent algebraic expressions in different

	A.APR.1 A.APR.3 A.CED.1 A.CED.2 A.REI.11 F.IF.4 F.IF.5 F.IF.7a F.IF.7c F.IF.8b F.BF.1b	forms and use algebra and graphs to determine whether they are equivalent. Lessons provide opportunities for students to identify linear, exponential, and quadratic functions using multiple representations. Lessons introduce students to the concept of building new functions on a coordinate plane by operating on separate functions.
4: Quadratic Functions	A.SSE.1a A.SSE.2 A.CED.1 A.APR.1 F.IF.4 F.IF.9 F.IF.7a F.BF.1a F.BF.3	This chapter begins with a matching and sorting activity to review the different forms of quadratic functions. Key characteristics of quadratic functions and graphs are identified. Lessons then provide opportunities for students to explore and identify transformations performed on a quadratic function to form a new function $(x) = Af(B(x - C)) + D$. This transformational function form is introduced in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of underlying function. In the later part of the chapter, lessons provide opportunities for students to explore and understand what conditions are necessary to write a unique quadratic function. The set of complex numbers is introduced and students will operate with the imaginary number i . Finally, students will solve quadratic functions over the set of complex numbers.
5: Polynomial Functions	A.SSE.1a A.SSE.1b A.APR.1 A.APR.3 F.IF.4 F.IF.5 F.IF.7a F.IF.7b F.IF.7c F.BF.3	This chapter begins with two different problem situations to explore how cubic functions are built. Lessons provide opportunities for students to connect characteristics and behaviors of cubic functions to their factors. An emphasis is placed on verifying equivalence between different forms both algebraically and graphically. Students will explore polynomial functions to gain an understanding of end behavior, symmetry, and whether a function is even, odd, or neither. Questions will ask students to graph, write, and explain the effects of transformations on cubic functions, and then draw conclusions about how symmetry is preserved in transformed functions. In the later part of the chapter, lessons focus on building various polynomial functions by operating with the basic power functions on a coordinate plane and in a table of values. Questions then ask students to compare and contrast the various polynomials to understand all the possible shapes and key characteristics for linear, quadratic, cubic, quartic,

		and quantic functions. At the end of the chapter, lessons focus on students' understanding that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.
6: Polynomial Expressions and Equations	A.SSE.1a A.SSE.3a A.CED.3 A.REI.11 A.APR.1 A.APR.2 A.APR.3 F.IF.4 F.IF.6 F.IF.8a N.CN.8	This chapter presents opportunities for students to analyze, factor, solve, and expand polynomial functions. The chapter begins with an analysis of key characteristics of polynomial functions and graphs. Lessons then provide opportunities for students to divide polynomials using two methods and to expand on this knowledge in order to determine whether a divisor is a factor of the dividend. In addition, students will solve polynomial equations over the set of complex numbers using the Rational Root Theorem. In the later part of the chapter, lessons provide opportunities for students to utilize polynomial identities to rewrite numeric expressions and identify patterns. Students will also explore Pascal's Triangle and the Binomial Theorem as methods to expand powers of binomials.
7: Polynomial Functions	A.CED.1 A.CED.2 A.CED.3 A.REI.11 F.IF.4 F.IF.5 F.IF.7b F.IF.9 F.BF.1 F.LE.3 S.ID.6a	This chapter provides opportunities for students to solve polynomial inequalities algebraically and graphically. Lessons present various problem situations and ask students to use a graphing calculator to determine the polynomial regression function that best models the data. Students then use their regression functions to answer questions. Piecewise functions are introduced for situations where a single polynomial function is not the most appropriate model for a set of data. At the end of the chapter, the lesson provides opportunities for students to compare properties of two functions each represented in a different way. Questions present functions that are represented using a graph, table of values, equation, or description of its key characteristics.
8: Sequences and Series	A.SSE.1a A.SSE.4 A.CED.1 F.BF.2	This chapter begins with a review of arithmetic and geometric sequences and their explicit and recursive formulas. Lessons provide opportunities for students to explore finite and infinite arithmetic series, and then finite and infinite geometric series are used to derive formulas to compute each type of series. Students will explore and analyze the common ratios of several infinite geometric series to understand under what conditions the series is either divergent or convergent. In the later part of the chapter, lessons provide opportunities for students to apply their understanding of

		geometric series to solve problems.
9: Rational Functions	F.IF.5 F.IF.8a F.BF.3 A.APR.6 A.SSE.2 A.CED.1 A.REI.2	This chapter presents opportunities for students to analyze, graph, and transform rational functions. The chapter begins with an analysis of key characteristics of rational functions and graphs. Lessons then expand on this knowledge for transformations of rational functions. Students will determine whether graphs of rational functions have vertical asymptotes, removable discontinuities, both, or neither, and then sketch graphs of rational functions detailing all holes and asymptotes. Finally, students will explore problem situations modeled by rational functions and answer questions related to each scenario.
10: Solving Rational Equations	A.SSE.2 A.APR.6 A.REI.2 A.REI.11 A.CED.1	This chapter provides opportunities for students to connect their knowledge of operations with rational numbers to operations with rational expressions. Lessons provide opportunities for students to analyze and compare the process to add, subtract, multiply, and divide rational numbers to the same operations with rational expressions. Students conclude rational expressions are similar to rational numbers and are closed under all the operations. In the later part of the chapter, lessons provide opportunities for students to write and solve rational equations and list restrictions. Student work is presented throughout the chapter to demonstrate efficient ways to operate with rational expressions and efficient ways to solve rational equations based on the structure of the original equation.
11: Radical Functions	F.IF.4 F.IF.5 F.IF.7b F.IF.9 F.BF.3 F.BF.4a A.REI.2 N.RN.1 N.RN.2	This chapter presents opportunities for students to explore radical functions, simplify radical expressions, and solve radical equations. The chapter begins with an introduction to radical functions as inverses of power functions. Students will graph radical functions, write their equations, and determine their key characteristics. Lessons then expand on this knowledge for transformations of radical functions. In the later part of the chapter, lessons provide opportunities for students to rewrite radicals using rational exponents and extract roots from radical expressions. Students will also multiply, divide, add, and subtract radical expressions. Finally, students will analyze solution strategies for radical equations, and solve real-world problem situations using radical equations.
12: Graphing Exponential and Logarithmic Functions	F.IF.4	This chapter presents opportunities for students to analyze, graph, and transform exponential and logarithmic functions. The chapter begins with an exploration of exponential functions. Students will

		<p>analyze key characteristics of exponential functions and graphs. Lessons then expand on this knowledge for transformations of exponential functions.</p> <p>In the later part of the chapter, lessons focus on logarithmic functions. Student will determine key characteristics of logarithmic functions and graphs. Students will also transform logarithmic functions and make generalizations about the effect of a transformation on an inverse function.</p>
13: Exponential and Logarithmic Equations	F.LE.4	<p>In this chapter, students use their understanding of exponential and logarithmic functions to solve exponential and logarithmic equations. Students begin by building understanding and fluency with exponential and logarithmic expressions, including estimating the values of logarithms on a number line and then use this understanding to derive the properties of logarithms. Students explore alternative methods for solving logarithmic equations and solve exponential and logarithmic equations in context.</p>
14: Modeling with Functions	F.IF.1b F.IF.2 F.IF.3 F.IF.4 F.IF.5 F.IF.7a-e F.BF.1a F.BF.1b F.BF.1c F.BF.2 F.BF.3 F.BF.4b F.LE.2 F.LE.5 A.CED.3 A.REI.12	<p>In this chapter, students explore various real-world and purely mathematical situations that are modeled with functions. Function composition is developed, and students apply function composition to solve contextual problems. Students also use functions to draw graphics, to model optimal solutions and self-similarity, and to study situations modeled by logistic growth, such as the spread of infectious diseases. Students end the chapter by choosing appropriate functions to model a variety of problem situations.</p>
15: Trigonometric Functions	F.TF.1 F.TF.2 F.TF.5 F.IF.7e	<p>This chapter begins with a problem situation involving a Ferris wheel in which students explore how periodic functions are built. Lessons provide opportunities for students to analyze the graphs of periodic functions for characteristics such as the maximum, minimum, period, amplitude, and midline. Students will explore the unit circle to understand radian measure and convert between angle measures in degrees and radians. Using new understanding of the unit circle, radian measure, and periodic functions, students will investigate the sine and cosine functions as well as their characteristics and graphs.</p> <p>In the later part of the chapter, students recall the</p>

		transformational function form $g(x) = Af(B(x - C)) + D$ to graph and analyze transformations of the sine and cosine functions and build a graph of the tangent function using a context. Students will analyze the characteristics of the tangent, secant, cosecant, and cotangent graphs, and apply their knowledge of transformations to sketch graphs of transformed tangent, secant, cosecant, and cotangent functions.
16: Trigonometric Equations	F.TF.1 F.TF.2 F.TF.5 F.TF.8	In this chapter, students are introduced to solving trigonometric equations. They use their knowledge of the unit circle, radian measures, and the graphical behaviors of trigonometric functions to solve sine, cosine, tangent, secant, cosecant, and cotangent equations. Students learn trigonometric identities including sum, difference, double and half angle, product and sum identities. Students then apply all that they have learned to model various situations with trigonometric functions, including circular motion. Finally, students explore the damping function and modeling with trigonometric transformations.
17: Polar Coordinates and Complex Numbers	N-CN 3 N-CN 4 N-CN 5 N-CN 6	In this chapter students are introduced to polar coordinates. They are introduced to the polar coordinate plane and graphs of polar equations. They learn how to convert from polar to rectangular coordinates. They learn the polar form of complex numbers and calculate the product and quotient of complex numbers in polar form. They also find powers and roots of complex numbers in polar form.
18: Vectors	N-VM 1 N-VM 2 N-VM 4a N-VM 4b N-VM 4c N-VM 5a N-VM 5b F-IF 10. F-IF 11.	In this chapter students are introduced to the geometry of vectors. They learn the algebraic properties of vectors and how vectors incorporate parametric equations. They also explore parallel and perpendicular vectors in the plane.

***See attachment for specific standards addressed.**

TEXTBOOKS AND RESOURCE MATERIALS:

Integrated Math III, A Common Core Math Program; Carnegie Learning, 2013
Adv. Mathematical Concepts; Glencoe/McGraw Hill

COMMON CORE STANDARDS TO BE ADDRESSED:

The content standards addressed in this course come from each of the conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

DISTRICT ESLRS TO BE ADDRESSED:

When students exit a secondary mathematics course, they will be:

- **Self-directed Learners**: who will be able to use notes and a textbook to assist them in continuing their learning outside of the classroom setting.
- **Efficient Communicators**: who can explain mathematical concepts to others and use mathematics to organize and explain data.
- **Quality Producers**: who understand the importance of neat, organized work that demonstrates their thinking and understanding of the solution they've formed to solve a problem.
- **Constructive Thinkers**: who are able to attack problems with organization, logic, and mathematical skills they've developed in a systematic fashion.
- **Collaborative Workers**: who can work in a variety of settings in culturally diverse groups. They will be able to form and use study groups to strengthen their own understanding in addition to providing the same service for classmates.
- **Responsible Citizens**: who accept the consequences of their actions and who demonstrate their understanding of their role in the learning process.

Appendix of Standards: (Taken from CDE website)

Mathematics III Standards

Number and Quantity

The Complex Number System N-CN

Use complex numbers in polynomial identities and equations. [Polynomials with real coefficients; apply N.CN.9 to higher degree polynomials.]

8. (+) Extend polynomial identities to the complex numbers.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Algebra

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions. [Polynomial and rational]

1. Interpret expressions that represent a quantity in terms of its context.
 - a. Interpret parts of an expression, such as terms, factors, and coefficients.
 - b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
2. Use the structure of an expression to identify ways to rewrite it.

Write expressions in equivalent forms to solve problems.

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Arithmetic with Polynomials and Rational Expressions A-APR

Perform arithmetic operations on polynomials. [Beyond quadratic]

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.¹

Note: (+) Indicates additional mathematics to prepare students for advanced courses.

¹ The Binomial Theorem may be proven by mathematical induction or by a combinatorial argument.

Rewrite rational expressions. [Linear and quadratic denominators]

- Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations A-CED

Create equations that describe numbers or relationships. [Equations using all available types of expressions, including simple root functions]

- Create equations and inequalities in one variable **including ones with absolute value** and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* CA
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
- Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Reasoning with Equations and Inequalities A-REI

Understand solving equations as a process of reasoning and explain the reasoning. [Simple radical and rational]

- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Represent and solve equations and inequalities graphically. [Combine polynomial, rational, radical, absolute value, and exponential functions.]

- Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Functions

Interpreting Functions F-IF

Interpret functions that arise in applications in terms of the context. [Include rational, square root and cube root; emphasize selection of appropriate models.]

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations. [Include rational and radical; focus on using key features to guide selection of appropriate type of model function.]

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
 - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
 - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Building Functions F-BF

Build a function that models a relationship between two quantities. [Include all types of functions studied.]

1. Write a function that describes a relationship between two quantities.
 - b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Build new functions from existing functions. [Include simple, radical, rational, and exponential functions; emphasize common effect of each transformation across function types.]

1. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
 - a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.*

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

M3

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology. [Logarithms as solutions for exponentials]

Mathematics III

4.1. Prove simple laws of logarithms. CA

4.2 Use the definition of logarithms to translate between logarithms in any base. CA

4.3 Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. CA

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

2.1 Graph all 6 basic trigonometric functions. CA

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. □

Geometry

Similarity, Right Triangles, and Trigonometry G-SRT

Apply trigonometry to general triangles.

9. (+) Derive the formula $A = 1/2 ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section.

3.1 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola and graph the equation. [In Mathematics III, this standard addresses only circles and parabolas.] CA

Geometric Measurement and Dimension G-GMD

Visualize relationships between two-dimensional and three-dimensional objects.

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry G-MG

Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Statistics and Probability

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable.

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Making Inferences and Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Using Probability to Make Decisions S-MD

Use probability to evaluate outcomes of decisions. [Include more complex situations.]

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Pre-Calculus Standards

The Complex Number System

N-CN Number

Perform arithmetic operations with complex numbers.

3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120° .*
6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Vector and Matrix Quantities

N-VM

Represent and model with vector quantities.

1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v , $|v|$, $\|v\|$, v).
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
4. Add and subtract vectors.
 - a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
 - b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
 - c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w , with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. Multiply a vector by a scalar.
 - a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
 - b. Compute the magnitude of a scalar multiple cv using $\|cv\| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).
10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to

produce another vector. Work with matrices as transformations of vectors.

12. Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.