

FOLSOM CORDOVA UNIFIED SCHOOL DISTRICT

Course Outline

AP CALCULUS BC

Date: January 2006

Subject Area: Mathematics

Proposed Grade Level(s): 12

Course Length: 2 Semesters/1 Year

Grading: A-F

Number of Credits: 5 Per Semester

Prerequisites: 'B' or better in AP Calculus (AB)

COURSE DESCRIPTION:

This course is a continuation of the Advanced Placement (AP) Calculus course, and is comparable to a second semester Calculus course in colleges and universities. Each student will complete a simulated AP exam in early May, which will be counted as approximately 25% of the final exam for the course. The course will include: techniques of integration, improper integrals, indeterminate forms, multiple integration, applications of integration, infinite series, parametric equations, polar integration, differential equations, and applications of hyperbolic functions.

GENERAL GOALS / PURPOSES:

A major objective of the class is to prepare students for the Advanced Placement Calculus B/C exam, to be given in May. Most universities award credit to students based upon their scores on this exam. Students will be encouraged to participate in the national exam.

STUDENT READING COMPONENT:

Students will receive instruction on the effective use of their textbook. This course includes applications where effective reading and analysis are taught as part of the course.

STUDENT WRITING/ORAL COMPONENT:

Students will have opportunities to express their understanding of concepts in writing, as well as presenting work orally in class discussions. All written work will follow standard rules of English. Any research projects will follow MLA format, which has been distributed at all secondary sites.

Final Assessment:

Final exams covering theory, operations, derivations, and applications will be given. In early May, a simulated AP Calculus BC exam will be given, and be weighted approximately 25% of the total final exam score.

DETAILED UNITS OF INSTRUCTION:

Note: Topics may be taught in different order and enriched by additional topics.

First Semester:

1) Applications of Integration

- a) Area of a specified region - Integration will be used to find the area between two curves and between intersecting curves. [16.0]
- b) Volumes of specified solids
 - i) The Disk Method - The volume of a solid of revolution can be found using the disk method. Integration can also be used to find the volume of a solid with known cross sections. [16.0]
 - ii) The Shell Method - The shell method uses cylindrical shells and offers advantages over the disk method. [16.0]
- c) Arc Length and Surfaces of Revolution - Integration and the use of a basic formula are used to find the arc length of a smooth curve and the area of a surface of revolution. [16.0]
- d) Work - Integration and basic principles of physics will be used to find the work done by a constant force and by a variable force. [16.0]
- e) Moments, Centers of Mass - Students will first understand the definition of mass, and then find the center of mass of a one-dimensional and a two-dimensional system. [14.0]
- f) Fluid Pressure and Fluid Force - Integration and the concept of buoyancy, are used to determine fluid pressure and fluid force on submerged surfaces. [14.0]
- g) The Catenary - Hyperbolic functions, and basic vectors and forces, are used to determine the equation of a hanging chain (catenary). Solving with graphic calculators will enable students to determine the sag, and span of a catenary. [14.0]

2) Integration Techniques

- a) Integration by Parts - This technique can be applied to a wide variety of functions, and is particularly useful for integrands involving products of algebraic and transcendental functions. [17.0, 20.0]
- b) Trigonometric Integrals - Trigonometric substitution can be used to evaluate integrals involving radicals by eliminating the radical in the integrand. [17.0, 18.0, 19.0, 20.0]
- c) Tables and Other Techniques - Functions that are not readily integratable can be integrated by the use of tables. Another method available is integration by parts. [17.0, 20.0]
- d) Indeterminate Forms and L'Hopital's Rule - Limits that are of indeterminate form (Numerator and denominator both with limits of zero or infinity) can be evaluated using L'Hopital's Rule. [22.0]
- e) Improper Integrals - Various improper integrals that have an infinite limit of integration can be evaluated. Some improper integrals that have an infinite discontinuity can also be evaluated. [22.0]

3) Differential Equations

- a) Separation of Variables - Students will recognize and solve differential equations in which all x terms can be collected with dx , and all y terms with dy . [27.0]
- b) Orthogonal Trajectories - Students will investigate problems involving a family of curves, each of which is orthogonal to all members of a given family of curves. [27.0]
- c) Applications
 - i) Electricity - Orthogonal trajectories can be applied to electrostatics, and differential equations in general can be applied to electrical networks. [27.0]
 - ii) Newton's laws of motion - Applying differential equations to Newton's laws of motion, will enable students to solve spring-mass problems, and problems involving harmonic motion. [27.0]

Second Semester:

4) Infinite Series

- a) Sequences - Students will list the terms of a sequence and then determine whether it converges or diverges. Students will also write a formula for the n th term of a sequence, and examine properties of monotonic and bounded sequences. [23.0]
- b) Series
 - i) Alternating series - Students use the Alternating Series Test to determine whether an infinite series converges, and use a remainder theorem to approximate the sum of an alternating series. [23.0]
 - ii) Ratio and Root tests – The Ratio and Root tests are used to determine convergence and divergence of a geometric series. [23.0]
 - iii) Harmonic series - The Harmonic Series, used in music, are examined using integral tests. [23.0]
 - iv) Taylor Polynomials - Techniques for finding polynomial approximations of elementary functions are investigated. [26.0]
 - v) Representation of functions by power series - Techniques for finding the radius and interval of convergence are discussed. [24.0]
 - vi) Taylor and MacLaurin series - Functions are expressed as Taylor and MacLaurin series, and these are differentiated and integrated. [26.0]
- 4) Parametric Equations - Curves are sketched by using a set of parametric equations. Techniques for eliminating the parameter are discussed.

5) Polar Forms -

- a) Polar Equations - Students will study the polar coordinate system and rewrite rectangular equations in polar form.
- b) Polar Coordinates and Polar Graphs - Techniques for graphing in polar form and for finding the slope of a tangent line to a graph in polar form are explored.
- c) Polar Integration - Polar integration is applied when rectangular integration is more cumbersome.
- d) Applications
 - i) Area in Polar Coordinates - Polar integration is used to determine the area of a region bounded by a polar graph.
 - ii) Arc Length in Polar Coordinates - The method of determining the arc length of a polar graph is examined.
 - iii) Other - Polar integration is used to find the surface area of a polar graph.

THIS COURSE WILL PREPARE STUDENTS FOR THE CAHSEE AND/OR THE FCUSD EXIT EXAMS:

Mathematics

LAB FEE, IF REQUIRED:

None

SUBJECT AREA CONTENT STANDARDS TO BE ADDRESSED:

- 14.0 Students apply the definition of the definite integral to model problems in physics, and economics, obtaining results in terms of integrals.
- 16.0 Students use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work.
- 17.0 Students compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution, integration by parts, and trigonometric substitution. They can also combine these techniques when appropriate.
- 18.0 Students know the definitions and properties of inverse trigonometric functions and the expression of these functions as definite integrals.
- 19.0 Students compute, by hand, the integrals of rational functions by combining the techniques in Standard 17.0 with the algebraic techniques of partial fractions and completing the square.

- 20.0 Students compute the integrals of trigonometric functions by using the techniques noted above.
- 22.0 Students understand improper integrals as limits of definite integrals.
- 23.0 Students demonstrate an understanding of the definitions of convergence and divergence of sequences and series of real numbers. By using such tests as the comparison test, ratio test, and alternate series test, they can determine whether a series converges.
- 24.0 Students understand and can compute the radius (interval) of the convergence of power series.
- 25.0 Students differentiate and integrate the terms of a power series in order to form new series from known ones.
- 26.0 Students calculate Taylor polynomials and Taylor series of basic functions, including the remainder theorem.
- 27.0 Students know the techniques of solution of selected elementary differential equations and their applications to a wide variety of situations, including growth-and-decay problems. Note: Other standards are all addressed in Calculus AB. Parametric and polar forms are included in the scope of Calculus BC, but are too advanced for the Content Standards.

DISTRICT ESLRs TO BE ADDRESSED:

Students will be:

- **Self-Directed Learners:** who will be able to use notes and a textbook to assist them in continuing their learning outside of the classroom setting.
- **Effective Communicators:** who can explain mathematical concepts to others and use mathematics to organize and explain data.
- **Quality Producers:** who understand the importance of neat, organized work that demonstrates their thinking and understanding of the solution they've formed to solve a problem.
- **Constructive Thinkers:** who are able to attack problems with organization, logic, and mathematical skills they've developed in a systematic fashion.
- **Collaborative Workers:** who can work in a variety of settings in culturally diverse groups. They will be able to form and use study groups to strengthen their own understanding in addition to providing the same service for classmates.
- **Responsible Citizens:** who accept the consequences of their actions, and who demonstrate their understanding of their role in the learning process.