TRIGONOMETRY

Date: January 2009
Proposed Grade Level(s): 9-12
Grading: A – F
Prerequisites: “C” or better in Algebra 2

BRIEF COURSE DESCRIPTION:
In this course students will expand on the basic trigonometry learned in Geometry in preparation for the study of calculus. Topics covered will include trigonometric functions and their graphs, inverse trigonometric functions, right and oblique triangles, identities, complex numbers and polar graphs. In addition, students will complete a unit on Probability and Statistics. At the end of the course students will have completed all the state standards for Trigonometry and Probability and Statistics. Following this course, students can advance to Math Analysis in preparation for Calculus.

GENERAL GOALS/PURPOSES:
The purpose of this course is to build a solid foundation of trigonometric skills and concepts that will be used in calculus and other advanced courses. As stated in the state standards, trigonometry uses the techniques that students have previously learned from the study of algebra and geometry. The trigonometric functions studied are defined geometrically rather than in terms of algebraic equations. The goal is to develop abstract thinking and mathematical reasoning through trigonometric proofs and applications.

STUDENT READING COMPONENT:
Students will receive instruction on the effective use of their textbook. Mathematical vocabulary will be a primary focus. This course includes applications where effective reading and analysis are taught as part of instruction.

STUDENT WRITING/ORAL COMPONENT:
Students will have opportunities to express their understanding of concepts in writing as well as present work orally to the class. All written work will follow standard rules of English. Any research projects will follow MLA format, which has been distributed at all secondary sites.

DETAILED UNITS OF INSTRUCTION:
1. Review Topics
   a. Real Numbers and their properties
   b. Solving Equations
   c. Graphs of Equations in the Cartesian Plane
   d. Linear Equations in two variables.
   e. Functions and their graphs
   f. Inverse Functions
   g. Distance formula
2. Trigonometric Functions
   a. Angles in the coordinate plane
   b. Angle measures in degrees and radians
   c. Angular and linear velocity
   d. Cosine and sine functions
   e. Other trigonometric functions
   f. Trig functions of special angles
   g. Evaluating trig functions

3. Graphing Trigonometric Functions
   a. Periodic functions and symmetry
   b. Graphs of sine and cosine functions
   c. Amplitude and period
   d. Phase shift and vertical shift
   e. Graphing by addition of ordinates
   f. Graphs of tangent and cotangent functions
   g. Graphs of secant and cosecant functions

4. Right Triangle Trigonometry and Basic Identities
   a. Solving right triangles
   b. Angles of elevation and depression
   c. Fundamental Identities: reciprocal, ratio, Pythagorean, odd-even
   d. Equivalent Trigonometric Expressions
   e. Proving Identities

5. Oblique Triangles
   a. Law of Sines
   b. Law of Sines: ambiguous case
   c. Law of Cosines
   d. Law of Tangents
   e. Area of a Triangle
   f. Heron’s formula
   g. Vectors in the plane

6. Trigonometric Identities
   a. Cosine: sum and difference identities
   b. Sine: sum and difference identities
   c. Tangent: sum and difference identities
   d. Double-angle identities
   e. Half-angle identities
   f. Product/sum identities

7. Inverse Trigonometric Functions
   a. Inverse sine and cosine functions
   b. Other inverse functions
   c. Solving trigonometric equations

8. Complex Numbers
   a. Polar coordinates
   b. Graphs of polar coordinates
   c. Sums and differences of complex numbers
   d. Products and quotients of complex numbers
   e. Complex numbers in polar form
f. Multiplying and dividing complex numbers in polar form

g. DeMoivre’s Theorem

h. Roots of complex numbers

9. Probability
   a. Binomial Theorem
   b. Counting Principles
   c. Sample Spaces
   d. Probability of an event
   e. Mutually Exclusive events
   f. Independent events
   g. Complement of an event

10. Statistics
    a. Standard distributions (normal, binomial, and exponential)
    b. Measures of central tendency
    c. Variance and Standard deviation
    d. Data displays – frequency tables, histograms, line and bar graphs, stem-and-leaf displays, scatterplots, box-and-whisker plots

Final Assessment:
Each high school will use departmental finals for each semester.

**SUBJECT AREA CONTENT STANDARDS TO BE ADDRESSED:**

**Trigonometry**

1.0 Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians.

2.0 Students know the definition of sine and cosine as y- and x- coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.

3.0 Students know the identity \( \cos^2 x + \sin^2 x = 1 \).
   3.1 Students prove that this identity is equivalent to the Pythagorean theorem.
   3.2 Students prove other trigonometric identities and simplify others by using the identity \( \cos^2 x + \sin^2 x = 1 \).

4.0 Students graph functions of the form \( f(t) = A \sin(Bt + C) \) or \( f(t) = A \cos(Bt + C) \) and interpret A, B, and C in terms of amplitude, frequency, period, and phase shift.

5.0 Students know the definitions of the tangent and cotangent functions and can graph them.

6.0 Students know the definitions of the secant and cosecant functions and can graph them.

7.0 Students know that the tangent of the angle that a line makes with the x-axis is equal to the slope of the line.

8.0 Students know the definitions of the inverse trigonometric functions and can graph the functions.

9.0 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.

10.0 Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities.

11.0 Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric identities.

12.0 Students use trigonometry to determine unknown sides or angles in right triangles.

13.0 Students know the law of sines and the law of cosines and apply those laws to solve problems.

14.0 Students determine the area of a triangle, given one angle and the two adjacent sides.

15.0 Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa.
16.0 Students represent equations given in rectangular coordinates in terms of polar coordinates.
17.0 Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form.
18.0 Students know DeMoivre’s theorem and can give nth roots of a complex number given in polar form.
19.0 Students are adept at using trigonometry in a variety of applications and word problems.

**Probability and Statistics**

1.0 Students know the definition of the notion of independent events and can use the rules for addition, multiplication, and complementations to solve for probabilities of particular events in finite sample spaces.
2.0 Students know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.
3.0 Students demonstrate an understanding of the notion of discrete random variables by using them to solve for the probabilities of outcomes, such as the probability of the occurrence of five heads in 14 coin tosses.
4.0 Students are familiar with the standard distributions (normal, binomial, and exponential) and can use them to solve for events in problems in which the distribution belongs to those families.
5.0 Students determine the mean and the standard deviation of a normally distributed random variable.
6.0 Students know the definitions of the mean, median, and mode of a distribution of data and can compute each in particular situations.
7.0 Students compute the variance and standard deviation of a distribution of data.
8.0 Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatterplots, and box-and-whisker plots.

**THIS COURSE WILL PREPARE STUDENTS FOR THE CAHSEE AND/OR CSTs IN:**

Math

**LAB FEE, IF REQUIRED:** None

**DISTRICT ESLRs TO BE ADDRESSED:**

Students will be:

- **Self-directed Learners** who will be able to use notes and a textbook to assist them in continuing their learning outside of the classroom setting.
- **Efficient Communicators** who can explain mathematical concepts to others and use mathematics to organize and explain data.
- **Quality Producers/Performers** who understand the importance of neat, organized work that demonstrates their thinking and understanding of the solution they’ve formed to solve a problem.
- **Constructive Thinkers** who are able to attack problems with organization, logic, and mathematical skills they’ve developed in a systematic fashion.
- **Collaborative Workers** who can work in a variety of settings in culturally diverse groups. They will be able to form and use study groups to strengthen their own understanding in addition to providing the same service for classmates.
- **Responsible Citizens** who accept the consequences of their actions and who demonstrate their understanding of their role in the learning process.