Inside Out
Triangle Sum, Exterior Angle, and Exterior Angle Inequality Theorems

Vocabulary
Write the term that best completes each statement.

1. The __________________________ states that the measure of an exterior angle of a triangle is greater than the measure of either of the remote interior angles of the triangle.
2. The __________________________ states that the sum of the measures of the interior angles of a triangle is 180°.
3. The __________________________ states that the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles of the triangle.
4. The __________________________ are the two angles that are non-adjacent to the specified exterior angle.

Problem Set
Determine the measure of the missing angle in each triangle.

1. \[ m\angle B = 180° - (78° + 37°) = 65° \]
List the side lengths from shortest to longest for each diagram.

7. \( m\angle C = 180^\circ - (48^\circ + 21^\circ) = 111^\circ \)

The shortest side of a triangle is opposite the smallest angle. So, the side lengths from shortest to longest are \( a, b, c \).
Name _______________________________ Date ____________

9. \[\angle L = 28^\circ, \angle M = 118^\circ\]

10. \[\angle X = 42^\circ, \angle Y = 84^\circ\]

11. \[\angle X = 67^\circ, \angle Y = 79^\circ, \angle Z = 64^\circ\]

12. \[\angle A = 50^\circ, \angle B = 30^\circ, \angle C = 90^\circ, \angle D = 60^\circ, \angle E = 90^\circ\]
Identify the interior angles, the exterior angle, and the remote interior angles of each triangle.

13. \( \triangle WXY \)
   - Interior angles: \( \angle XYZ, \angle YZX, \angle XZY \)
   - Exterior angle: \( \angle WXZ \)
   - Remote interior angles: \( \angle XYZ, \angle YZX \)

14. \( \triangle RST \)
   - Interior angles: \( \angle RTS, \angle STR, \angle RTS \)
   - Exterior angle: \( \angle RST \)
   - Remote interior angles: \( \angle RTS, \angle STR \)

15. \( \triangle EFG \)
   - Interior angles: \( \angle EFG, \angle FGE, \angle GEF \)
   - Exterior angle: \( \angle EGH \)
   - Remote interior angles: \( \angle EFG, \angle FGE \)

16. \( \triangle ABC \)
   - Interior angles: \( \angle ABC, \angle BCA, \angle CAB \)
   - Exterior angle: \( \angle ACD \)
   - Remote interior angles: \( \angle ABC, \angle BCA \)

17. \( \triangle JKM \)
   - Interior angles: \( \angle JKL, \angle KJL, \angle LJK \)
   - Exterior angle: \( \angle JKL \)
   - Remote interior angles: \( \angle JKL, \angle KJL \)

18. \( \triangle QRS \)
   - Interior angles: \( \angle QSR, \angle QRS, \angle RQS \)
   - Exterior angle: \( \angle QSR \)
   - Remote interior angles: \( \angle QSR, \angle QRS \)
LESSON 3.1 Skills Practice

Name ____________________________ Date __________

Solve for $x$ in each diagram.

19. \[ m\angle GFH = 180^\circ - 130^\circ = 50^\circ \]

\[ m\angle GHK = m\angle GFH + m\angle FGH \]

\[ 99^\circ = 50^\circ + x \]

\[ 49^\circ = x \]

20. \[ m\angle GFH = 180^\circ - 130^\circ = 50^\circ \]

\[ m\angle GHK = m\angle GFH + m\angle FGH \]

\[ 99^\circ = 50^\circ + x \]

\[ 49^\circ = x \]

21. \[ m\angle IHJ = 2x \]

\[ m\angle JHK = 81^\circ \]

22. \[ m\angle URV = 64^\circ \]

\[ m\angle RSV = 90^\circ \]

\[ (x + 8)\]
Use the given information for each triangle to write two inequalities that you would need to prove the Exterior Angle Inequality Theorem.

25. Given: Triangle $RST$ with exterior $\angle TRQ$
   Prove: $m \angle TRQ > m \angle S$ and $m \angle TRQ > m \angle T$

26. Given: Triangle $QRS$ with exterior $\angle PQR$
   Prove:
Name ____________________________________________  Date __________

27. 
Given: Triangle $UVW$ with exterior $\angle TUV$
Prove: 

28. 
Given: Triangle $GHJ$ with exterior $\angle FGJ$
Prove: 

29. 
Given: Triangle $LMN$ with exterior $\angle KLN$
Prove: 

30. 
Given: Triangle $ABC$ with exterior $\angle BCD$
Prove: 

Trade Routes and Pasta, Anyone?
The Triangle Inequality Theorem

Vocabulary
Identify an example of each term in the diagram of triangle \( ABC \).

1. Triangle Inequality Theorem

Problem Set
Without measuring the angles, list the angles of each triangle in order from least to greatest measure.

1. \( \triangle GHF \)

The smallest angle of a triangle is opposite the shortest side. So, the angles from least to greatest are \( \angle H, \angle F, \angle G \).
Determine whether it is possible to form a triangle using each set of segments with the given measurements. Explain your reasoning.

7. 3 inches, 2.9 inches, 5 inches  
   Yes. A triangle can be formed because the sum of the two shortest sides is greater than the longest side.  
   Sum of the Two Shortest Sides: 3 + 2.9 = 5.9  
   Longest Side: 5

8. 8 feet, 9 feet, 11 feet  

9. 4 meters, 5.1 meters, 12.5 meters  

10. 7.4 centimeters, 8.1 centimeters, 9.8 centimeters
Name ___________________________________________ Date __________

11. 10 yards, 5 yards, 21 yards  
12. 13.8 kilometers, 6.3 kilometers, 7.5 kilometers

13. 112 millimeters, 300 millimeters, 190 millimeters  
14. 20.2 inches, 11 inches, 8.2 inches

15. 30 cm, 12 cm, 17 cm  
16. 8 ft, 8 ft, 8 ft

Write an inequality that expresses the possible lengths of the unknown side of each triangle.

17. What could be the length of $\overline{AB}$?

18. What could be the length of $\overline{DE}$?

$AB < AC + BC$

$AB < 10$ meters $+ 8$ meters

$AB < 18$ meters
19. What could be the length of $HI$?

20. What could be the length of $JL$?

21. What could be the length of $MN$?

22. What could be the length of $QR$?
Stamps Around the World
Properties of a $45^\circ$–$45^\circ$–$90^\circ$ Triangle

Vocabulary
Define the term in your own words.

1. $45^\circ$–$45^\circ$–$90^\circ$ Triangle Theorem

Problem Set
Determine the length of the hypotenuse of each $45^\circ$–$45^\circ$–$90^\circ$ triangle. Write your answer as a radical in simplest form.

1. $\sqrt{2}$ in.

2. $\frac{5\sqrt{2}}{2}$ cm

3. $9\sqrt{2}$ ft

4. $7\sqrt{2}$ km

The length of the hypotenuse is $2\sqrt{2}$ inches.
Determine the lengths of the legs of each $45^\circ$–$45^\circ$–$90^\circ$ triangle. Write your answer as a radical in simplest form.

5. $a = \sqrt{2} = 16$
   \[a = \frac{16}{\sqrt{2}}\]
   \[a = \frac{16\sqrt{2}}{2\sqrt{2}}\]
   \[a = \frac{16\sqrt{2}}{2} = 8\sqrt{2}\]
   The length of each leg is $8\sqrt{2}$ centimeters.

6. $a = \sqrt{2} = 12$

7. $a = \sqrt{2} = 6\sqrt{2}$ ft

8. $a = \sqrt{2} = 8\sqrt{2}$ m

Use the given information to answer each question. Round your answer to the nearest tenth, if necessary.

9. Soren is flying a kite on the beach. The string forms a $45^\circ$ angle with the ground. If he has let out 16 meters of line, how high above the ground is the kite?
   \[a = \frac{16}{\sqrt{2}}\]
   \[a = \frac{16\sqrt{2}}{2\sqrt{2}}\]
   \[a = \frac{16\sqrt{2}}{2} = 8\sqrt{2} \approx 11.3\]
   The kite is approximately $11.3$ meters above the ground.
10. Meena is picking oranges from the tree in her yard. She rests a 12-foot ladder against the tree at a 45º angle. How far is the top of the ladder from the ground?

11. Emily is building a square bookshelf. She wants to add a diagonal support beam to the back to strengthen it. The diagonal divides the bookshelf into two 45º–45º–90º triangles. If each side of the bookshelf is 4 feet long, what must the length of the support beam be?

12. Prospect Park is a square with side lengths of 512 meters. One of the paths through the park runs diagonally from the northeast corner to the southwest corner, and it divides the park into two 45º–45º–90º triangles. How long is that path?
Determine the area of each triangle.

13. \[ a = 16 \text{ mm} \]

\[ a\sqrt{2} = 16 \]

\[ a = \frac{16}{\sqrt{2}} \]

\[ a = \frac{16\sqrt{2}}{2\sqrt{2}} \]

\[ a = \frac{16\sqrt{2}}{2} \]

\[ a = 8\sqrt{2} \]

The area of the triangle is 64 square millimeters.

14. \[ a = 18 \text{ in.} \]

15. \[ a = 7 \text{ ft} \]
Use the given information to answer each question.

17. Eli is making a mosaic using tiles shaped like 45°–45°–90° triangles. The length of the hypotenuse of each tile is 13 centimeters. What is the area of each tile?

\[
\begin{align*}
a \sqrt{2} &= 13 \\
a &= \frac{13}{\sqrt{2}} = \frac{13\sqrt{2}}{\sqrt{2}\sqrt{2}} \\
a &= \frac{13\sqrt{2}}{2} \\
A &= \frac{1}{2} \left( \frac{13\sqrt{2}}{2} \right) \left( \frac{13\sqrt{2}}{2} \right) \\
A &= \frac{169}{8} = 21.125 \\
A &= \frac{169}{4} = 42.25
\end{align*}
\]

The area of each tile is 42.25 square centimeters.

18. Baked pita chips are often in the shape of 45°–45°–90° triangles. Caitlyn determines that the longest side of a pita chip in one bag measures 3 centimeters. What is the area of the pita chip?
19. Annika is making a kite in the shape of a 45°–45°–90° triangle. The longest side of the kite is 28 inches. What is the area of the piece of fabric needed for the kite?

20. A tent has a mesh door that is shaped like a 45°–45°–90° triangle. The longest side of the door is 36 inches. What is the area of the mesh door?
Construct each isosceles triangle described using the given segment.

21. Construct right isosceles triangle $ABC$ with segment $BC$ as the hypotenuse by constructing $45^\circ$ angles at $B$ and $C$.

22. Construct right isosceles triangle $WXY$ with segment $WX$ as the hypotenuse by constructing $45^\circ$ angles at $W$ and $X$. 
23. Construct right isosceles triangle $PQR$ with $RQ$ as a leg and $\angle R$ as the right angle.

24. Construct right isosceles triangle $DEF$ with $DF$ as a leg and $\angle D$ as the right angle.
More Stamps, Really?
Properties of a 30°–60°–90° Triangle

Vocabulary
Write the term that best completes each statement.

1. The __________ states that the length of the hypotenuse in a 30°–60°–90° triangle is two times the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

Problem Set
Determine the measure of the indicated interior angle.

1. \( m \angle ABC = 60^\circ \)

2. \( m \angle DFE = \)

3. \( m \angle HAK = \)

4. \( m \angle TRA = \)
Given the length of the short leg of a $30^\circ$–$60^\circ$–$90^\circ$ triangle, determine the lengths of the long leg and the hypotenuse. Write your answers as radicals in simplest form.

5. \[ \sqrt{3} \text{ ft} \]

6. \[ 5 \text{ in.} \]

\[ a = 3 \text{ feet} \]
\[ b = 3\sqrt{3} \text{ feet} \]
\[ c = 2(3) = 6 \text{ feet} \]

7. \[ \sqrt{6} \text{ mm} \]

8. \[ \sqrt{15} \text{ cm} \]

Given the length of the hypotenuse of a $30^\circ$–$60^\circ$–$90^\circ$ triangle, determine the lengths of the two legs. Write your answers as radicals in simplest form.

9. \[ 20 \text{ m} \]

10. \[ 16 \text{ km} \]

\[ c = 20 \text{ meters} \]
\[ a = \frac{20}{2} = 10 \text{ meters} \]
\[ b = 10\sqrt{3} \text{ meters} \]

11. \[ 6\sqrt{3} \text{ yd} \]

12. \[ 4\sqrt{2} \text{ ft} \]
Given the length of the long side of a 30°–60°–90° triangle, determine the lengths of the short leg and the hypotenuse. Write your answers as radicals in simplest form.

13. $b = 8\sqrt{3}$ inches
   $a = \frac{8\sqrt{3}}{\sqrt{3}} = 8$ inches
   $c = 2(8) = 16$ inches

14. $b = 11\sqrt{3}$ meters
   $a = 11\sqrt{3}$ meters
   $c = 2a = 2(11\sqrt{3}) = 22\sqrt{3}$ meters

15. $b = 12$ miles
   $a = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ miles
   $c = 2a = 2(4\sqrt{3}) = 8\sqrt{3}$ miles

16. $b = 18$ feet
   $a = \frac{18}{\sqrt{3}} = 6\sqrt{3}$ feet
   $c = 2a = 2(6\sqrt{3}) = 12\sqrt{3}$ feet

Determine the area of each 30°–60°–90° triangle. Round your answer to the nearest tenth, if necessary.

17. $a = \frac{6}{2} = 3$ centimeters
   $b = 3\sqrt{3}$ centimeters
   $A = \frac{1}{2} \cdot 3 \cdot 3\sqrt{3}
   A = \frac{9\sqrt{3}}{2} \approx 7.8$ square centimeters

The area of the triangle is approximately 7.8 square centimeters.
18. \[ \triangle \text{Diagram with angle } 60^\circ \text{ and side } 12 \text{ km} \]

19. Universal Sporting Goods sells pennants in the shape of 30º–60º–90º triangles. The length of the longest side of each pennant is 16 inches.

20. A factory produces solid drafting triangles in the shape of 30º–60º–90º triangles. The length of the side opposite the right angle is 15 centimeters.
Construct each triangle described using the given segment.

21. Construct a $30^\circ$–$60^\circ$–$90^\circ$ triangle by first constructing an equilateral triangle with $\overline{MN}$ as a side and then bisecting one of the sides.

22. Construct a $30^\circ$–$60^\circ$–$90^\circ$ triangle $RST$ by first constructing an equilateral triangle with $\overline{RS}$ as a side and then bisecting the angle at $R$. 
23. Construct a 30°–60°–90° triangle $EFG$ with $EF$ as the side opposite the 30° angle by first constructing an equilateral triangle.

24. Construct a 30°–60°–90° triangle $ABC$ by first copying angle $A$ and then drawing $AB$ as the hypotenuse.